Multi-Attribute Vickrey Auctions when Utility Functions are Unknown

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Abstract

Multi-attribute auctions allow negotiations over multiple attributes besides price. For example in task allocation, service providers can define their service by means of multiple attributes, such as quality of service, deadlines, or delay penalties. Auction mechanisms assume that the players have evaluation functions over the space of attributes that assign a single value to any combination of attributes. This value (or cost) is directly comparable to price. We argue that in some situations, some of the attributes are difficult to convert to cost, e.g., in transportation it is often important which driver is going to deliver a given truckload. Such personal preferences of a customer are difficult to quantify.

To allow negotiations over such non-monetary attributes we relax the requirement of universally comparable utility functions, and give an incentive-compatible auction mechanism that uses only preference orders of the parties and not globally comparable function values. The suggested mechanism assumes that the bidders and the auctioneer have individual total orders over the space of possible contracts, but no utility functions. Each bidder places its bids using its own order, and the winner is chosen by the auctioneer's order. The actual attribute values are chosen based on the second-best bid. It is shown that this Vickrey intuition yields an incentive-compatible mechanism.

1 Introduction

Multi-attribute auctions extend traditional price-only negotiations to negotiations over price and other attributes. One example is procurement, when multiple suppliers compete over different quality attributes (e.g. color, or warranty) and price. In such an environment, decisions are based on scoring functions that express the values of different attribute combinations in money (cost functions), which can be simply compared to price to express utility. In some cases, however, it is not possible express all attributes in terms of costs, even when it is possible to compare two choices. In a task allocation setting, the auctioneer may value a contract with an old customer more than one with an unknown one, but there is no natural way to express this in money. Our goal is to find an auction mechanism, where the bidders are best off by bidding the equivalent of their private value (incentive compatibility), and that can be used in case cost functions are unknown, or difficult to determine.

One of the most well-known incentive compatible mechanisms is the Vickrey auction [11]. Vickrey auctions are second-price sealed-bid auctions, where the optimal strategy is to bid one's true valuation. The original mechanism has been generalized in different ways. The Generalized Vickrey Auction [5, 10] is a mechanism to solve a rather complex combinatorial auctioning problem where multiple divisible goods are to be allocated to multiple bidders. Another generalization is when multiple attributes of a certain indivisible good are considered. For example Che [1] has introduced multi-dimensional auctions, where bids consist of values of multiple attributes, including price. In his approach every party has a scoring function that assigns a value to a combination of attributes expressing his own valuation (costs, preferences, etc.). Bidders place bids based on their own scoring function, and the auction mechanism selects the bidder with the highest score according to the auctioneer's scoring function. Che showed that his proposed mechanism, where the contracted values of the attributes (including price) are determined by the second-best combination of attribute-values is incentive compatible.

Following Che's footsteps, Parkes and Kalagnanam [6, 7] have proposed iterative mechanisms that are incentive compatible, efficient, and allow iterative revelation of private values.

While Che focused on buyer-optimality, Parkes et al. concentrated on market-efficient mechanisms. David et al. [2] studied the multi-attribute auction as well. They considered a first-score sealed-bid and four different English auction mechanisms, and analyzed the applicability of these mechanisms in different environments, providing optimal strategies for bidders and the auctioneer. Especially, they have suggested optimal scoring rules for the auctioneer, and have shown under which conditions it is optimal to share the truth with the bidders.

In some cases, however, it may be difficult to compute scores for different attribute values. The evaluation of scoring functions might include lengthy computations (e.g. predictions), or the quantification of certain attributes might not make much sense (e.g. color). One example in task allocation could be, when the negotiation includes the tool to be used to execute the task. It can make a difference for the auctioneer if tool A or tool B is used (as part of the quality of service), but he may not be able to assign a value to the different configurations. A practical example from transportation is that certain customers prefer certain truck drivers over others. They might be friends, or a long term trust relationship could already have developed between them, or simply because the driver speaks the language of the customer. In such cases it is very difficult for the parties to assign values to different combinations of attribute values, even though they clearly have a preference order over them.

Either when the auction mechanism optimizes buyer's utility, or market efficiency, it is commonly assumed that the sellers' evaluations are comparable, since they are always expressed as costs (for a good review on auctions in artificial intelligence research see [9]). We relax this assumption and require the existence only of a total order over the attribute-value combinations. This is different from the commonly used assumption in that it does not require the existence of a universally comparable value (cost). We show for a multi-attribute case that auctions can be conducted even if bidders and the auctioneer cannot assign values to different attribute combinations (Section 2). We describe a 'multi-attribute no-value' version of the Vickrey mechanism (Section 3), and prove that it is incentive compatible (Section 4). The main properties of the proposed mechanism are then discussed in Section 5, and a conclusion wraps our contribution up (Section 6).

2 Auctioning Without Values

Let us consider the auctioning of tasks. The result of such an auction is a contract on the execution of the auctioned task. A contract consists of n attributes (such as price, quality, etc.) of continuous or discrete values.

$$C(a_1, \ldots, a_n)$$

Suppose, that both the m bidders and the auctioneer have a total ordering over the different combinations of attribute configurations: $(A^n, \preceq_a)$ for the auctioneer and $(A^n, \preceq_{b_j})$ for bidder j, $\forall j = 1..m$. Note that in a traditional multi-dimensional auction setting everybody would assign a value to every combination of attribute values. This would implicitly define the ordering, but would also imply that it is possible to convert every combination of attributes to a utility for all bidders.

Having only a total order over the attribute values means (see Figure 1) that on presentation of two combination of attribute values every party can tell his preferred one, although he cannot assign a value to it. Certain combinations can be considered equal, belonging to an equivalence class.

Equivalence classes represent such combinations of attribute values that are equally good for the given player. If one combination of attributes is chosen then the players can still offer other combinations in the same equivalence group for the opposing players to choose from.

Definition 1. The *equivalence class* of $(a_1, \ldots, a_n)$ for agent j consists of all combinations of attribute values that are equivalent with $(a_1, \ldots, a_n)$ for agent j.

$$[(a_1, \ldots, a_n)_j]_{\preceq_j} = \{(a'_1, \ldots, a'_n) | (a'_1, \ldots, a'_n) =_j (a_1, \ldots, a_n)\}$$

Besides having an order over combinations of attribute values $(A^n, \preceq)$, players have separate orderings over most individual attribute values too ($(A_1, \preceq_{A_1})$, where A_1 is the set of values attribute 1 can take on). They may prefer higher or lower prices, better or worse qualities, etc. It is reasonable to assume that a combination of preferred attributes is preferred over a combination of less preferred attributes (monotonicity).

$$a_1 \preceq_{A_1} a'_1 \wedge \ldots \wedge a_n \preceq_{A_n} a'_n \Rightarrow (a_1, \ldots, a_n) \preceq (a'_1, \ldots, a'_n) \quad (1)$$

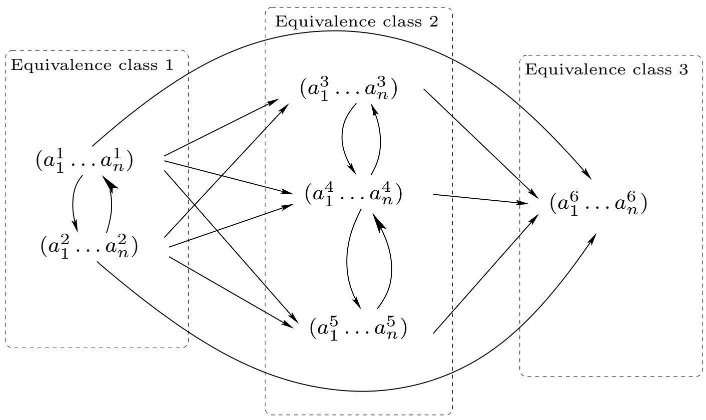

Figure 1: Total order over the combinations of n attributes

Another reasonable assumption to make is that the auctioneer's and the bidders' preference over the individual attributes are opposing. If the auctioneer tries to achieve a contract with low price and high quality, then the bidders will want high price and low quality. This implies that the bidders ordering over the individual attributes are the same (but the ordering of combinations is usually different for each bidder).

Bids consist of combinations of attributes ($bid_j = (a_1, \dots, a_n)_j$). Bidders have to determine every attribute before they can submit their bid. The chosen combination, or alternatively any other in the same equivalence group is their bid. The auctioneer collects all bids, and selects the most preferred one by the auctioneer. The bidder that submitted the most preferred bid is the winner.

For every bidder and every attribute there is a value, which is called the private value (a_{ij}^p is the private value of bidder j for attribute i). This value represents a threshold, below which the bidder does not want to win the auction. In case of price, the private value is the cost, in quality it is a basic quality level the bidder support anyway, etc. The combination of private values define a special equivalence class that we call the *private class*. The bidder is better off not winning the auction with a contract with attribute combination that is less preferred than his private class.

How to determine the values of the attributes for bidding depends on the auction mechanism. In an ascending auction bidders start from highly preferred values for every attribute and during consecutive bids, they move towards less preferred values following some bidding-strategy. They stop when the combination of attribute values belongs to the private class. In our proposed multi-attribute auction mechanism (see Section 3), the contract attributes are determined by the equivalence class of the second-best bid. This resembles the Vickrey mechanism, and indeed we will show (in Section 4) that the bidders best option is to place a bid from their private class.

3 The Mechanism

In the original Vickrey mechanism the bidders' optimal strategy is to bid their true valuation. When tasks are on auction and bids represent prices bidders ask to perform the tasks, the lowest bid is the winner, and the second-lowest bid determines the price. In such a situation agents' bid should be their cost. If their bid is higher, they reduce their chance of winning the auction but the price (determined by the second-lowest bid) remains the same. If their bid is lower than their cost then they have a higher chance of winning the auction, but the excess chance is exactly the chance that the price will not cover their cost.

Our mechanism is based on the recognition that the outcome of the auction does not depend on the values the auctioneer/bidders assign to attribute-value combinations. It only depends on how they order these combinations. Thus, it is not required to have an evaluation function over the attributes, but everybody has to have a total order over them.

The following steps are included in our mechanism, independent of the type of auction that is used.

1. Bidders $j \in \{1 \dots m\}$ select a combination of attribute values that they intend to bid (bid_j). Since it is equivalent to any other bid from the same equivalence class, we represent

their bid by the whole class.

$$[bid_j]_{\preceq_{b_j}} \tag{2}$$

2. A set of best bids (one for each agent j) is selected by choosing the most preferred bid by the auctioneer (using $\preceq_a$) in every equivalence class that was bidden.

$$\{\widehat{bid_j} | \widehat{bid_j} = \max_{\preceq_a} [bid_j]_{\preceq_{b_j}}, j = 1..m\} \tag{3}$$

3. The best of the bests is selected as winner.

$$bid_{win} = \max_{\preceq_a} (\widehat{bid_1}, \ldots, \widehat{bid_m}) \tag{4}$$

4. The contracted attributes are determined by the second-best bid (bid_{2nd}). It is equivalent for the auctioneer to contract the exact attribute-values in bid_{2nd}, or to contract any other value combination that belong to the same equivalence class.

$$bid_{2nd} \in [bid_{2nd}]_{\preceq_a} \tag{5}$$

5. The best combination of attribute values regarding the winner's ordering in $[bid_{2nd}]_{\preceq_a}$ is selected as contract attributes.

$$C(a_1, \ldots, a_n) = \max_{\preceq_{win}} [bid_{2nd}]_{\preceq_a} \tag{6}$$

The mechanism is backed by the same underlying thoughts as the single-issue Vickrey auction. The benefit of the winner bidder is defined by his competitors, which results in small benefit in a highly competitive market, and higher benefit, if the competition is not so tough.

The mechanism can be executed in four different ways regarding the private information of the parties. If the auctioneer's ordering is public, then the selection in step 2 can be executed by the bidders and they can submit their best bid regarding the auctioneer's ordering. If it is not public, then the bidders have to submit their whole equivalence class (Equation 2), and the auctioneer has to determine the best ones. Similarly, step 5 can be executed by the auctioneer, if the winner bidder shares his ordering with him. Otherwise, the auctioneer has to offer the whole equivalence class (eq_a) to the winner, who has to select the best combination of attribute-values himself.

The best performance in terms of communication is achieved, if no ordering is private. In this case, the amount of data to be transmitted is minimal, since the bidders can reduce their bids to one combination of attributes, and the auctioneer himself can determine the winner and the contract-values too. On the other hand, if all ordering information is private, whole sets of bids have to be communicated by the bidders, and the auctioneer also has to send his equivalence class to the winner to determine the contract-values. The minimal private information that has to be shared between the bidders and the auctioneer is the equivalence classes of the bids. The alternatives, where only one side has private information represent a trade-off between communication costs and information sharing.

4 Optimal Strategy

In a single-issue Vickrey auction bidders are best off bidding their 'true value'. In case tasks are on auction and bids represent prices bidders ask for performing the given task, the 'true value' to bid is the cost of performing the task. Bidders are better off by not winning the auction for a price that does not cover their costs.

In the proposed multi-attribute auctioning mechanism all combinations of attribute-values are ordered by the auctioneer ($\preceq_a$) and by the bidders ($\preceq_{b_j}, j = 1..m$) separately. Every bidder has a special value for every attribute, which is his private value regarding the given attribute (a_i^p is the private value of attribute i). In case the auction would only be about attribute i, values that are worse than a_i^p would not be acceptable for the bidder. In the multi-attribute case the equivalence class of bidder j that contains $a_i^p, i = 1..n$ is called the *private class* and any solution that is less preferred by $\preceq_{b_j}$ is not acceptable for the bidder.

Definition 2. The *private class* of bidder j is the equivalence class that contains the attribute combination, where every attribute takes its private value.

$$(a_1^p, \ldots, a_n^p)_j \in [bid_j^p]_{\preceq_{b_j}} \tag{7}$$

The private class contains minimal solutions for the bidder. He is better off not winning contracts with attribute-combinations that are less preferred than the private class. Note that single attribute values can be worse than the bidder's private value for that attribute, but the other attributes have to compensate for it.

Theorem 1. *In a multi-attribute second-price sealed-bid auction mechanism where the bidders' and the auctioneer's total ordering over the attribute values are diametrically opposed in the attribute space and are monotone in the individual attributes, the bidders are best off placing the bids in their private class.*

Proof. Bidders have two options other than bidding from their private class.

- Suppose a bidder places a bid that is preferred by him over bids in his private class:
  - Suppose he wins the auction. The actual attribute-values of the contract are determined by the second-best bid. That is the same as when he would have bidden his private class. Hence it is not beneficial to place a more preferred bid than those in the private class.
  - Suppose he looses the auction. There are two cases:
    - If the winning bid is better for the auctioneer than any bid from the bidder's private class, he would have lost the auction anyway.
    - If the winning bid is worse for the auctioneer than the best bid from his private class, then he could have won the auction by bidding truthfully.
- Suppose he places a bid that is less preferred by him than the ones in his private class:
  - Suppose he wins the auction. There are two cases:
    - If the second-best bid is less preferred by the auctioneer than the best bid in his private class, then the contracted values will be the same as when he would have bidden his true value.
    - If the second-best bid is more preferred by the auctioneer than the best bid in his private class, then he would have lost the auction by bidding truthfully. Though he wins the auction, the contracted combination of attribute-values are less preferred by him than the ones in his private class, which means that he would be better off by not winning the auction at all (see Equation 7).
  - Suppose he looses the auction. He would have lost the auction even if he had bid his private class.

In all possible outcomes, the bidder is not better off than by bidding from his private class. This proves that every bidder's best strategy to bid from their private class. □

5 Mechanism properties

Mechanism design is the problem of how to design the (social) rules of an interaction such that a some objective function is maximized even if the players are intended to handle only in their own interest [8]. Jackson [3] describes the most important characteristics of such social choice functions. The desirable equilibrium properties of a multi-attribute auction include: efficiency, buyer-optimality, individual-rationality, and budget-balance.

The mechanism described in this paper offers a dominant strategy for the bidders, which is to place a bid from their private class. Let us suppose that the auctioneer also has a private equivalence class, and that he does not make a contract with less preferred (by $\preceq_a$) attribute values than this class. Then the mechanism has the following properties in equilibrium.

Efficiency: An efficient algorithm allocates a good to the one who values it most. Analogously, an efficient algorithm allocates a task to the one who executes it for the cheapest price. The ultimate goal is to maximize the difference between the buyer's and the seller's valuation. In our case the traditional definition of efficiency does not apply, due to the lack of universally comparable valuation functions. One cannot tell which bidder prefers the buyer most. Our multi-attribute mechanism allocates the task to the one whose offer has the best combination of attributes (for the auctioneer). Thus it implements a locally efficient allocation from the buyer's point of view.

Buyer-optimality: Buyer optimality in task allocation means that the auctioneer has to pay the least possible amount for the winner to execute his task. Just as the single issue Vickrey auction, our mechanism is not buyer-optimal. The winner bidder would execute the task even if a less preferred contract (from the winner's point of view) was made. Generally it is not possible for a mechanism to be efficient and buyer-optimal at the same time [4].

Individual-rationality: A mechanism is individually rational for the players if they are not better off not participating in the mechanism at all. In a traditional single or multi-attribute auction, individual rationality means that neither the auctioneer nor the bidders have negative utility. In our mechanism the auctioneer and the winner have to prefer the contract over their private classes for the mechanism to be individually rational. In equilibrium, the bidders place no worse bids then their private class, and the contracted values will be no less preferred by the winner than his bid. On the other hand, the auctioneer may not sell the task with a contract worse than his private class. Thus individual rationality holds.

Budget-balance: In a single-issue (money) case the payment made by the buyer should equal the payment received by the seller for this property to hold. This can be generalized to multi-attribute case, where such a 'balance' is required in every attribute. In our case the attribute values are fixed in the contract: the seller provides the exact contracted values to the buyer.

Though the original definition of efficiency is not applicable in case of non-monetary attributes, the mechanism implements efficient outcome in a local sense, from the auctioneer's point of view. No best bidder could be selected for the auctioneer than the actual winner, but from the market's point of view there could be a better choice. Thus local efficiency may not express the same objective measure as the original one. If a locally efficient allocation was an efficient allocation in case we could express the value of the attributes in money is an open question.

However, local efficiency is enough for the mechanism to be ex-ante individually rational in equilibrium. If every bidder places a bid from his private class, then the mechanism ensures that the outcome will be as good for the winner as his bid. Any bidder can safely participate in the auction, as long as he bids from his private class, he cannot be worse off than not participating at all.

6 Conclusions and Future Research

Auctions are a popular form of distributed rational decision making. Several mechanisms have been developed for allocating a single task to a single server, multiple goods to multiple buyers, or a single good with multiple attributes to a single buyer. At the design of such mechanisms, the usual assumption to make is that the bidders and the auctioneer have a utility function over the space of the single or multiple attributes. This utility function implicitly expresses the players' preferences by assigning a value to the different outcomes.

In the current study we have relaxed this assumption, and have shown how multi-attribute auctions can be conducted when no utility value can be assigned to combinations of attribute values. We argue that this is important because it can capture situations with numerically not expressible attributes, which were impossible to capture before. We have suggested an incentive-compatible mechanism to allocate a single task with multiple attributes to a single server based only on preference orders. The mechanism implements a locally efficient outcome in an equilibrium, and it is individually rational for the players.

To extend the theory from total orders to partial orders is an interesting direction for further research. Besides, more investigation is required to determine the communicational needs of the mechanism. Currently it is not clear, what is the best way to represent and communicate equivalence classes based on the parties' orderings. Another future assignment is to implement the mechanism in artificial agents to enable them to negotiate over numerically not (easily) expressible attributes. One possible application is in multi-agent logistics, where truck agents and container agents negotiate over price, decommitment-penalty, deadlines, etc. Such bilateral multi-attribute negotiations render logistics to be a promising application domain.

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